

Word-Automata Translation

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1 Preliminaries

1.1 Automata

We consider finite-state automata over ω -words (hereafter “words”). An alphabet Σ is a finite nonempty set, and a *word* over Σ is an infinite sequence $w = w(0) \cdot w(1) \cdot \dots$ of letters in Σ . An *automaton* is a tuple $\mathcal{A} = \langle \Sigma, Q, \delta, Q_0, \alpha \rangle$, where the first four elements define its *structure* and α is its *acceptance condition*. In the structure, Σ is the input alphabet, Q is a finite set of states, $\delta : Q \times \Sigma \rightarrow 2^Q$ is a transition function, and $Q_0 \subseteq Q$ is a set of initial states. The acceptance condition is defined below.

Intuitively, the transition $\delta(q, \sigma)$ provides the set of states that the automaton may move to when it is in the state q and reads the letter σ . An automaton may have several initial states and its transition function may specify several target states for some state and letter, thus the automaton is *nondeterministic*. In the case that $|Q_0| = 1$ and for every $q \in Q$ and $\sigma \in \Sigma$, we have $|\delta(q, \sigma)| \leq 1$, the automaton is *deterministic*.

A *run* $r = r(0), r(1), \dots$ of $\mathcal{A}$ on $w = w(0) \cdot w(1) \cdot \dots \in \Sigma^\omega$ is an infinite sequence of states such that $r(0) \in Q_0$, and for every $i \geq 0$, we have $r(i+1) \in \delta(r(i), w(i))$. Note that a deterministic automaton has at most a single run on an input word, while a nondeterministic automaton may have several runs on the same word.

Acceptance is defined with respect to the set $\text{inf}(r)$ of states that the run r visits infinitely often. Formally, $\text{inf}(r) = \{q \in Q \mid \text{for infinitely many } i \in \mathbb{N}, \text{ we have } r(i) = q\}$. As Q is finite, it is guaranteed that $\text{inf}(r) \neq \emptyset$. The run r is *accepting* iff the set $\text{inf}(r)$ satisfies the acceptance condition α , and it is *rejecting* otherwise. Several acceptance conditions are defined in Section 1.2.

An automaton accepts a word if it has an accepting run on it. The language of an automaton $\mathcal{A}$, denoted by $L(\mathcal{A})$, is the set of words that $\mathcal{A}$ accepts. We also say that $\mathcal{A}$ *recognizes* the language $L(\mathcal{A})$. Two automata, $\mathcal{A}$ and $\mathcal{A}'$, are *equivalent* if $L(\mathcal{A}) = L(\mathcal{A}')$.

1.2 Automata Classes

The *class* of an automaton characterizes its branching mode (deterministic or nondeterministic) and its acceptance condition. Several acceptance conditions are studied in the literature; the main ones are:

- *Büchi*, where $\alpha \subseteq Q$, and r is accepting iff $\text{inf}(r) \cap \alpha \neq \emptyset$. It is named after Julius Richard Büchi, who introduced it in [Büc62].
- *co-Büchi*, where $\alpha \subseteq Q$, and r is accepting iff $\text{inf}(r) \cap \alpha = \emptyset$.
- *weak* is a special case of the Büchi condition, where every strongly connected component of the automaton is either contained in α or disjoint to α ; that is, no strongly connected component has a state in α and some other state not in α . The weak condition, with respect to alternating automata, was introduced in [MSS86].
- *parity*, where $\alpha = \{S_1, S_2, \dots, S_{2k}\}$ with $S_1 \subset S_2 \subset \dots \subset S_{2k} = Q$, and r is accepting if the minimal index i for which $\text{inf}(r) \cap S_i \neq \emptyset$ is even. The parity condition is often equivalently viewed as a mapping from states to non-negative integers, called *priorities* (or *colors*), and a run is accepting if the maximal priority visited infinitely often is even. The parity condition was first defined in [Mos84].

- *Rabin*, where $\alpha = \{\langle B_1, G_1 \rangle, \langle B_2, G_2 \rangle, \dots, \langle B_k, G_k \rangle\}$, with $B_i, G_i \subseteq Q$ and r is accepting iff for some $i \in [1..k]$, we have $\text{inf}(r) \cap B_i = \emptyset$ and $\text{inf}(r) \cap G_i \neq \emptyset$. It is named after Michael Rabin, who introduced it in [Rab69].
- *Streett*, where $\alpha = \{\langle B_1, G_1 \rangle, \langle B_2, G_2 \rangle, \dots, \langle B_k, G_k \rangle\}$, with $B_i, G_i \subseteq Q$ and r is accepting iff for all $i \in [1..k]$, we have $\text{inf}(r) \cap B_i = \emptyset$ or $\text{inf}(r) \cap G_i \neq \emptyset$. It is named after Robert Streett, who introduced it in [Str81].
- *Muller*, where $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_k\}$, with $\alpha_i \subseteq Q$ and r is accepting iff for some $1 \leq i \leq k$, we have that $\text{inf}(r) = \alpha_i$. It is named after David Muller, who introduced it in [Mul63].

A run that is not accepting is *rejecting*. Notice that Büchi and co-Büchi are special cases of the parity condition, which is in turn a special case of both the Rabin and Streett conditions. In the latter conditions, we refer to the B_i and G_i sets as the “bad” and “good” sets, respectively.

The number of sets in the parity and Muller acceptance conditions or pairs in the Rabin and Streett acceptance conditions is called the *index* of the automaton. For weak, co-Büchi, and Büchi automata, the index is 1.

We denote the different classes of automata by three letter acronyms in $\{D, N\} \times \{W, B, C, P, R, S, M\} \times \{W\}$. The first letter stands for the branching mode of the automaton (deterministic or nondeterministic); the second for the acceptance-condition (weak, Büchi, co-Büchi, parity, Rabin, Streett, or Muller); and the third indicates that the automaton runs on words. For example, DBW stands for deterministic Büchi automata on words.

By $\gamma[k]$, we denote the subclass of γ with index k . For example, NRW[1] stands for nondeterministic Rabin automata with a single acceptance pair.

We say that a language L is γ -recognizable or γ -realizable if L can be recognized by an automaton in the class γ .

1.3 Typeness

Type. For two automata classes, γ and γ' , for example DCW and DBW, we say that γ is γ' -type if whenever a γ automaton $\mathcal{A}$ has an equivalent γ' -automaton $\mathcal{B}$, there is also an equivalent γ' -automaton $\mathcal{C}$ over the structure of $\mathcal{A}$. For example, DBW is DCW-type since whenever a DBW has an equivalent DCW, there is also an equivalent DCW over the structure of the original DBW. More details on typeness can be found in [KPB94, KMM04].

Natural typeness

Some acceptance conditions are more detailed than others. For example, the Büchi condition can obviously be viewed as a special case of the Rabin condition. Thus, DBW is obviously DRW-type and NBW is obviously NRW-type. This situation, in which an automata class γ is γ' -type since the acceptance condition of γ' is more detailed, is called *natural typeness*.

The weak acceptance condition is, by definition, a special case of the Büchi condition. Furthermore, since in a weak automaton the strongly connected components are either contained in α or disjoint to α , it is also a special case of the co-Büchi condition. Note

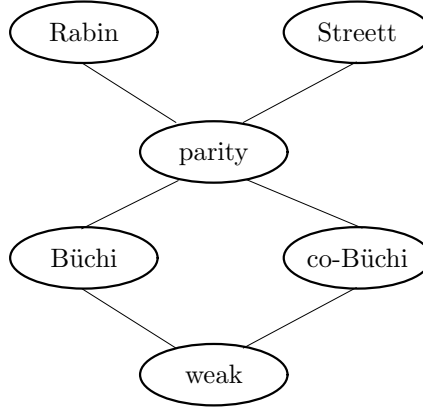


Figure 1: The natural-typeness hierarchy.

that the acceptance set α' of the equivalent co-Büchi automaton should be dual to the acceptance set α of the weak automaton. That is, $\alpha' = Q \setminus \alpha$.

Both the Büchi and the co-Büchi conditions can be viewed as a special case of the parity condition. A Büchi condition α is equivalent to the parity condition $\{\emptyset, \alpha, Q, Q\}$, and a co-Büchi condition α is equivalent to the parity condition $\{\alpha, Q\}$.

The parity condition can be viewed as a special case of both the Rabin and the Streett conditions: A parity condition $\{\alpha_1, \alpha_2, \dots, \alpha_{2k}\}$ is equivalent to the Rabin condition $\{\langle \alpha_{2k-1}, \alpha_{2k} \rangle, \dots, \langle \alpha_3, \alpha_4 \rangle, \langle \alpha_1, \alpha_2 \rangle\}$ and to the Streett condition $\{\langle \alpha_{2k-1}, \alpha_{2k-2} \rangle, \dots, \langle \alpha_3, \alpha_2 \rangle, \langle \alpha_1, \emptyset \rangle\}$.

The Muller condition provides all possible information with respect to the set $\text{inf}(r)$ of states that a run r visits infinitely often. Hence, every automaton whose acceptance condition is based on $\text{inf}(r)$ has an equivalent Muller automaton on the same structure. However, as opposed to the above cases, the index of the equivalent Muller automaton might be exponentially larger [Saf89, Bok17a]. Hence, we do not consider the other automata types as having natural typeness to Muller automata.

The natural-typeness hierarchy is depicted in Figure 1.

1.4 Expressive Power

Automata on infinite words recognize ω -regular languages, yet not all of the above classes have the same expressive power.

With the Büchi condition, NBW recognize all ω -regular languages [McN66], while DBWs are strictly less expressive [Lan69].

With the co-Büchi condition, neither NCW nor DCW recognize all ω -regular languages [McN66]. Unlike the Büchi condition, co-Büchi automata do not gain extra power from nondeterminism, having that $\text{NCW} = \text{DCW}$ [MH84]. Furthermore, DCW is dual to DBW, meaning that a language L is in DBW iff its complement is in DCW. Indeed, by viewing a DBW $\mathcal{A}$ as a DCW $\mathcal{A}'$, one gets an automaton for the complementing language, and vice versa.

With the weak condition, nondeterminism does add power, having that $\text{NWW} \supsetneq \text{DWW}$. Furthermore, $\text{NWW} = \text{NCW}$, while $\text{DWW} = \text{DBW} \cap \text{DCW}$ [Lan69, BJW01].

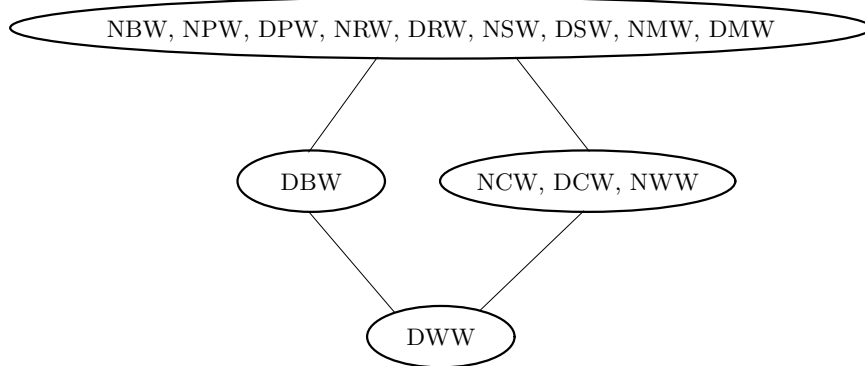


Figure 2: The expressive-power hierarchy.

As for Rabin, Streett, parity, and Muller automata, both the deterministic and non-deterministic classes recognize all ω -regular languages [Tho90].

The expressive-power hierarchy is illustrated in Figure 2.

It should be noted that different classes with the same expressive power might still have a significant difference in their succinctness. For example, there is a family of languages L_n , $n = 1, 2, \dots$, such that for every $n \in \mathbb{N}$ there is a DSW $\mathcal{A}_n$ of size n recognizing L_n , while the size of every NBW recognizing L_n is at least 2^n [SV89].

1.5 Automaton Size

The *size* of an automaton is the maximum of its element sizes, namely the maximum of the alphabet length, the number of states, the number of transitions, and the index, where the latter denotes the size of the acceptance condition. Out of the four elements, the number of states and the index are the dominant ones. For Büchi automata the index is 1, for parity it can be as large as the number of states, and for Rabin, Streett, and Muller it can be exponentially larger than the number of states.

When analyzing translations between automata, the first measure to consider is the size blowup, while a second consideration is the influence of and on each of the automaton elements. For example, a nondeterministic Büchi automaton of size n can be translated to a deterministic Rabin automaton of size in $2^{O(n \log n)}$. The constructed Rabin automaton has $2^{O(n \log n)}$ states. As for its index, by some algorithms it is in $O(n)$ [Saf89], and by others it is in $2^{O(n)}$ [Sch09]. The latter algorithm produces a bigger index, while allowing for a slightly smaller number of states (which is still in $2^{O(n \log n)}$).

In some cases, as the one mentioned above, the size blowup and state blowup coincide. Yet, there are cases in which they are very different. For example, the translation of Streett to Rabin automata involves a state blowup in $\Theta(n^2)$ and a size blowup in $2^{\Theta(n)}$, whereas the translation of Rabin to Streett automata involves a state blowup in $2^{\Theta(n)}$ and a size blowup in $\Theta(n^2)$ [Bok17b]. The difference between the size blowup and state blowup stems from the tradeoff between the number of states and the index—in translating Rabin to Streett there is an exponential state blowup and no index blowup, whereas from Streett to Rabin there is a quadratic state blowup and an exponential index blowup.

In addition to the number of states and the index, the alphabet length, which might be exponentially larger than the number of states, should be considered. The common practice is to provide the upper bounds for arbitrary alphabets and to seek lower bounds with fixed alphabets. For example, in the Büchi-determinization construction mentioned above, Michel provided a matching lower bound of $2^{\Omega(n \log n)}$ [Mic88], however he used Büchi automata over alphabets of length linear in the number of states (and therefore with quadratically many transitions). Afterwards, Löding improved the lower bound to be over a fixed alphabet [Löd99]. An often productive approach is to start with a rich alphabet of length exponential in the number of states, get a state lower bound, while ignoring the alphabet length and the number of transitions, and then enhance it to be over a fixed alphabet [Yan08, BKR10]. The challenge in this approach is to encode the rich alphabet by a fixed one without enlarging the number of states.

Less central is the number of transitions, which might be as large as the product of the alphabet length and quadratically the number of states. The influence of the number of transitions on the size blowup tends to go hand in hand with the influence of the number of states.

2 Decision Problems

The best possible complexity for the nonemptiness problem is NLOGSPACE and linear time, taking its lower bound from the reachability problem. It is indeed achieved with Büchi automata. For the stronger acceptance conditions, except for Streett, it remains in NLOGSPACE, while exceeding the linear time, and for Streett it is PTIME-complete.

The best possible complexity for the universality problem on nondeterministic automata is PSPACE-complete, taking its lower bound from automata on finite words. A possible way to perform the universality check of an automaton $\mathcal{A}$ of the considered types is the following: translate $\mathcal{A}$ to a Streett automaton $\mathcal{B}$ with only a polynomial size blowup, then complement $\mathcal{B}$ to a Büchi automaton $\mathcal{C}$ on the fly, having a potential exponential space, and check the nonemptiness of $\mathcal{C}$ in logarithmic space, yielding a PSPACE algorithm in the size of $\mathcal{A}$ [Saf89].

The complexity of the nonemptiness and universality problems (of nondeterministic automata) is summarized in Table 1.

Table 1: Decision Problems

The complexity of the nonemptiness and universality checks of nondeterministic automata. The complexity is with respect to the automaton's size, and if specified, with respect to m states and index k . (See Section 1.5 for size definition.)

Checks of Nondeterministic	Nonemptiness	Universality
Weak	Linear time, NL-complete [CES86, VW94]	PSPACE-comp. [Saf89]
Co-Büchi		
Büchi		
Parity	$O(m \log k)$ time, NL-comp. [EL87, KKV01]	
Rabin	$O(mk)$ time, NL-comp. [VW94]	
Streett	PTIME-comp. [EL87]	
Muller	NL-comp. [CBK90]	

3 Boolean Operations

In the nondeterministic setting, boolean operations on the classic automata types, except for Muller, roughly involve an asymptotically optimal size blowup: linear for union, quadratic for intersection, and singly exponential for complementation. These blowups are inevitable already in automata over finite words. In the deterministic setting, however, the picture is different, having an exponential size blowup on union or intersection for all of the classic ω -regular-complete types. Table 2 presents the size blowup involved in boolean operations.

Table 2: Boolean Operations

The size blowup involved in boolean operations. (See Section 1.5 for size definition.)

Operations Size Blowup	On Deterministic Automata			On Nondeterministic Automata		
	Union	Intersect.	Complement.	Union	Intersect.	Complement.
Weak	Quadratic		No blowup	Linear	Quad.	$2^{\Theta(n)}$ [MH84] (if possible)
Co-Büchi			No blowup [KMM04] (if possible)			$2^{\Theta(n \log n)}$ [Mic88, Saf89] [CZ11b]
Büchi						
Parity	Exponential [LY08, Bok18]		No blowup		Quad. - Quartic	
Rabin	Quad. [Bok18]	Exp. [Bok18]	Exp. [Löd99]		Quad. [Bok18]	$2^{\Theta(n^2 \log n)}$ [KV05, CZL09] [CZ11a]
Streett	Exp. [Bok18]	Quad. [Bok18]				
Muller	Exp. [Bok18]		Exp. [Saf89]			Exp. [Bok18]

4 Translations: High-Level Tables

Table 3 presents a high-level view of the size blowup involved in automata translations, while Table 4 considers the state blowup, ignoring the size of the index, which might be exponential in the number of states.

- The capital letters in the tables abbreviate Weak, Co-Büchi, Büchi, Parity, Rabin, Streett, and Muller automata.
- Translations between some automata classes are not always possible (see Section 1.4).
- Self translations are not marked and are obviously in $\Theta(n)$.
- A question mark stands for an exponential gap between the currently known lower and upper bounds.

Table 3: Size Blowup

Translating an automaton of size n results in an automaton of size as described in the table. (See Section 1.5 for size definition.)

From	To	Deterministic							Non-Deterministic						
		W	C	B	P	R	S	M	W	C	B	P	R	S	M
Det.	W	.							.						
	C		.							.					
	B			.											
	P				.										
	R		$\Theta(n^2)$						$\frac{O(n^2)}{\Omega(n)}$		$\Theta(n^2)$	$\frac{O(n^2)}{\Omega(n)}$	.	$\frac{O(n^2)}{\Omega(n)}$	
	S			$\Theta(n^2)$			$2^{\Theta(n \log n)}$	.	$\Theta(n)$		$2^{\Theta(n)}$			.	
	M		$\frac{O(n^2)}{\Omega(n)}$	?			$\frac{2^{O(n \log n)}}{2^{\Omega(n)}}$	.	$\frac{O(n^2)}{\Omega(n)}$		$\Theta(n^3)$		$\Theta(n^2)$	.	
Non-Det.	W								.						
	C		$2^{\Theta(n)}$							.					
	B														
	P														
	R								$2^{\Theta(n)}$						
	S														
	M														

*¹: The upper bounds range between $2^{2^{O(n)}}$ and $2^{2^{O(n^3 \log n)}}$.

*²: The lower bound to DBW is $2^{\Omega(n \log n)}$.

*³: The lower bound to DBW is $2^{\Omega(n)}$.

Table 4: State blowup

Translating an automaton with n states (and index as large as possible) results in an automaton with the number of states as described in the table.

To		Deterministic						Non-Deterministic							
From		W	C	B	P	R	S	M	W	C	B	P	R	S	M
Det.	W	$\Theta(n)$						$\Theta(n)$							
	C														
	B														
	P	$2^{\Theta(n)}$						$\Theta(n^2)$							
	R														
	S														
	M														
		$2^{\Theta(n)}$		$2^{\Theta(n \log n)}$		$\Theta(n)$		$2^{\Theta(n)}$		$\Theta(n^2)$					
		$2^{\Theta(n)}$													
Non-Det.	W	$2^{\Theta(n)}$						$\Theta(n)$							
	C														
	B														
	P	$2^{\Theta(n \log n)}$						$\Theta(n^2)$							
	R														
	S														
	M														
		?		$2^{2^{\Theta(n)}}$		$2^{\Theta(n)}$									
		* ⁴		$2^{O(n^2 \log n)}$											
		?		$2^{2^{\Theta(n)}}$						$\Theta(n^2)$					

*⁴: Tight to DPW and DRW; The lower bound to DBW, DSW, and DMW is $2^{\Omega(n \log n)}$.

5 Deterministic to Deterministic

5.1 All deterministic $\rightarrow$ DWW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

Construction details

The weak acceptance condition is the weakest one (which we consider here), and all the other conditions are, indeed, more expressive. Yet, when the translation is possible, it can be done over the original structure of the automaton [KMM04].

5.2 All deterministic $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: n states and index in $2^{\Theta(n)}$ [Saf89, Bok17a]

Construction details

All of our deterministic automata classes have equivalent DMWs over the same state space. Yet, the Muller index might be exponential in its number of states, listing all possible subsets.

Lower bound details

The high descriptive level of the Muller condition has its cost in the index. In [Saf89], Safra shows a family of languages L_n that can be recognized by NBWs with n states, whereas an equivalent DMW must have an index in $\Omega(2^{2^n})$. Safra's languages, however, use an alphabet that is exponential in the number of states. Hence, size wise, the result does not provide an exponential lower bound.

In [Bok17a], Safra's result is strengthened, considering a family L_n of deterministic looping automata (Büchi automata whole of whose states are accepting) over a fixed alphabet, and showing that an equivalent DMW must have an index in $\Omega(2^n)$. Thus, we get the exponential size blowup from all the considered automata classes.

5.3 DCW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

5.4 DBW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

5.5 DPW $\rightarrow$ DCW and DBW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KPB94].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

Construction details

A parity condition can be viewed as a special case of both a Rabin and a Streett conditions (see Section 1.3). The former is DBW-type [KPB94] and the latter is DCW-type, by dualizing the result of [KPB94].

5.6 DRW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Bok17b]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

It shown in [Bok17b] that every NCW-recognizable DRW and DMW with n states, m transitions, and index k can be translated to an equivalent DCW $\mathcal{C}$ with nk states and mk transitions. The construction idea is as follows.

The NCW-recognizable DRW or DMW with n states and index k is first translated to an equivalent NCW, as per the translation in [BK12] of an NRW or NMW to an NCW. In general, the constructed NCW might have $kn2^n$ states. However, we analyze the special case in which the translated automaton is deterministic, and show that the constructed NCW has up to kn states.

The next step is to determinize the constructed NCW. In general, co-Büchi determinization is done via the breakpoint (Miyano-Hayashi) construction, and might result in an exponential state blowup [MH84, BKR10]. Yet, we analyze the constructed NCW to be of a special form, a union of DCWs over the same structure, for which we provide a different determinization construction that introduces no state blowup.

Lower bound details

Size lower bound. It is shown in [Bok17b] that for every $n \in \mathbb{N}$, there is a DBW-recognizable DSW over a two-letter alphabet with $2n+1$ states, $3n$ transitions, and index n , for which equivalent DBWs have at least $n^2/2$ states. By the duality of DRWs and DSWs and of DBWs and DCWs, the result follows to the translation of DRWs to DCWs.

State lower bound. It is shown in [Bok17b] that for every $n \in \mathbb{N}$, there is an NCW-recognizable DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NCWs have at least 2^n states.

5.7 DRW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KPB94].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

5.8 DRW and DSW $\rightarrow$ DPW, DRW and DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$

Upper bound: [GH82]

Lower bound: [Löd99]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $O(n2^{k \log k})$ states and index in $\Theta(k)$ [Saf89]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(n)$

Upper bound: [GH82]

Lower bound: [Löd99]

Construction details

The construction is known as the *Index Appearance Records* (IAR), based on the *Last Appearance Records* (LAR) technique. It was used by Safra in [Saf89, Lemma 1.2] for translating a DSW to a DRW. The generated DRW already fits the parity acceptance condition, having $2k + 2$ priorities as explained below, thus providing the $\text{DSW} \rightarrow \text{DPW}$ translation. Due to the duality of DSW and DRW and the self duality of DPW, we get a translation of $\text{DRW} \rightarrow \text{DPW}$.

In Safra's proof of [Saf89, Lemma 1.2], he constructs a DRW whose states are tuples, in which two of the elements are r and g , both having values in $[1..k + 1]$; and the Rabin accepting pairs are (B_i, G_i) for $i \in [1..k + 1]$, where each set B_i consists of all states in which $i > r$ and each set G_i consists of all states in which $g = i$. Observe that for every $i < j \in [1..k + 1]$, we have $B_i \subseteq B_j$ and $G_i \neq G_j$. We may thus define the corresponding parity priorities as follows:

Priority	States satisfying
$2k + 2$	$g = 1$
$2k + 1$	$g = 2; r = 1$
$2k$	$g = 2; r > 1$
$2k - 1$	$g = 3; r \leq 2$
$2k$	$g = 3; r > 2$
$\vdots$	$\vdots$
1	$g = k + 1; r \leq k$
0	$g = k + 1; r = k + 1$

Once k is in $\Omega(n)$, one may view the original automaton as a DMW and convert it to a DPW. The DPW would have up to $2^{O(n \log n)}$ states and an index up to $O(n)$ (see Section 5.13).

Lower bound details

In [Löd99, Theorem 7], there is a family of DSWs over a fixed alphabet with n states and $k \in O(n)$, such that equivalent DRWs require $2^{\Omega(n \log n)}$ states. By the duality of DRWs and DSWs, the lower bound also stands for the translation of DRW to DSW.

5.9 DSW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KPB94, KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

5.10 DSW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Bok17b]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

By the duality of DRWs and DSWs and of DBWs and DCWs, it is the same as the translation of DRW to DCW (see Section 5.6).

5.11 DMW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $O(n^2)$ [Bok17b]

Lower bound: $\Omega(n)$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: nk states [Bok17b]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

It is shown in [Bok17b] that every NCW-recognizable DRW and DMW with n states, m transitions, and index k can be translated to an equivalent DCW $\mathcal{C}$ with nk states and mk transitions (see Section 5.6).

Lower bound details

Size lower bound. There is no non-trivial bound.

State lower bound. By the lower bound of translating a DRW to DCW (see Section 5.6).

5.12 DMW $\rightarrow$ DBW

*The size blowup involved in the translation is an **open problem**.*

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n)}$ [BK12]

Lower bound: $\Omega(n^2)$ [Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

The translation of an NMW to a DCW involves a state blowup of $2^{O(n)}$, for every index k (see Section 6.25). For translating a DMW to a DBW, one can first dualize the DMW, with no state blowup (but with possibly an index blowup), getting a DMW for the complement language. The latter can be translated to a DCW, as above, which in turn should be dualized to the required DBW.

Lower bound details

Size lower bound. [Bok17a, Theorem 9] provides the lower bound for translating a DMW to an NRW, and therefore also to a DRW, and since the considered languages are DBW-recognizable, the lower bounds also holds for the translation to DBW. (A DRW of a DBW-recognizable language can be translated to an equivalent DBW over the same structure.)

State lower bound. Since a DMW can be complemented with no state blowup, the state lower bound for $\text{DMW} \rightarrow \text{DCW}$ (see Section 5.11) holds also for $\text{DMW} \rightarrow \text{DBW}$. (Since the index of a DMW might grow in complementation, the duality does not work for the size blowup.)

5.13 DMW $\rightarrow$ DPW, DRW and DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n \log n)}$ [GH82]

Lower bound: $2^{\Omega(n)}$ [Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(n \log n)}$ states and index in $O(n)$ [GH82]

Lower bound: $2^{\Omega(n)}$ states [Bok17a]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(n)$ [GH82, Löd99]

Construction details

The construction is known as the *State Appearance Records* (SAR) [GH82], based on the *Last Appearance Records* (LAR) technique. It translates a DMW with n states to a DPW with $2^{O(n \log n)}$ states, regardless of the index k .

Lower bound details

In [Löd99], there is a $2^{\Omega(n \log n)}$ state and size lower bound for the translation of a DSW to a DRW. As every DSW has an equivalent DMW over the same state space, we get, state wise, a $2^{\Omega(n \log n)}$ lower bound. Yet, the family of DSWs that is used in the lower bound requires a Muller index that is exponential in the number of states. Hence, size wise, this result does not provide an exponential lower bound.

In [Bok17a], the lower bound proofs use families of DMWs over a fixed alphabet with $O(n)$ states and index in $O(n)$, for which equivalent DRWs and DSWs need $2^{\Omega(n)}$ states.

6 Nondeterministic to Deterministic

6.1 All nondeterministic $\rightarrow$ DWW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: 2^n [Sta83, MS97].

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: 2^n states [Sta83, MS97].

Construction details

The main idea is that the Myhill-Nerode theorem stands for DWW-recognizable languages [Sta83, MS97]. That is, there is a minimal DWW with a single state for every right-congruence equivalence class. Thus, the determinization of a non-deterministic automaton that is DWW-recognizable can be done via the subset construction, exactly as determinizing automata on finite words.

6.2 NWW $\rightarrow$ DCW, DPW, DRW, and DSW

A translation is **always possible**.

State blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states

Upper bound: $\frac{2}{3}3^n$ [MH84]

Lower bound: 2^n As any determinization.

Note that the tightness is in the exponent but not in the base, meaning that it is not known to be $\Theta(2^n)$.

Construction details

The translation to a DCW can be done via the *breakpoint construction* [MH84], also known as the Miyano-Hayashi construction (see Section 6.5). DCWs can be viewed as a special case of DPWs, DRWs and DSWs (see Section 1.3).

Open

For determinizing an NCW to a DCW, the breakpoint construction is known to be optimal [BKR10], however it is not known yet whether it is also optimal for translating an NWW to an equivalent DCW.

6.3 NWW and NCW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

State blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $\Theta(2^n)$ states [KMM04].

Construction details

By [KMM04], the target automaton can be built on top of the subset construction of the

original automaton.

6.4 NWW and NCW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{2^{\Theta(n)}}$ [MH84, Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states and index in $2^{2^{\Theta(n)}}$ [MH84, Saf89, Yan08, Bok17a]

Construction details

One can translate first the NWW or NCW to a DCW (see Section 6.5), getting $2^{O(n)}$ states [MH84]. The next translation of a DCW to a DMW (see Section 5.2) keeps the DCW's structure and only enlarges the index. The maximal Muller index may be exponential in its number of states, listing all possible subsets.

Lower bound details

Size lower bound. Corresponds to the index lower bound.

State lower bound. A $2^{\Omega(n)}$ state blowup is obvious for any determinization.

Index lower bound. In [Saf89], Safra shows a family of languages L_n that can be recognized by NBWs with n states, whereas an equivalent DMW must have an index in $\Omega(2^{2^n})$. Safra's languages, however, use an alphabet that is exponential in the number of states. Hence, size wise, the result does not provide an exponential lower bound.

In [Bok17a], Safra's result is strengthened, considering a family L_n of nondeterministic looping automata (Büchi automata whole of whose states are accepting) over a fixed alphabet, and showing that an equivalent DMW must have an index in $2^{2^{\Omega(n)}}$. Thus, we get the doubly exponential size blowup from all the considered automata classes.

6.5 NCW $\rightarrow$ DCW, DPW and DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(3^n)$ [MH84]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $\frac{2}{3}3^n$ states and index in $\Theta(1)$ [MH84, BKR10]

Construction details

The translation can be done via the *breakpoint construction* [MH84], also known as the Miyano-Hayashi construction. The construction of [MH84] does more than translating an NCW to a DCW, as it translates an alternating Büchi automaton (ABW) to an NBW. Given an NCW $\mathcal{A}$ for a language L , one can dualize it in $O(1)$ to a universal Büchi automaton (UBW) $\mathcal{B}$ for the complement language $\overline{L} = \Sigma^\omega \setminus L$. Since UBW is a special case of ABW, once can use the breakpoint construction for translating $\mathcal{B}$ to an

equivalent NBW B' . Moreover, a deeper look at the breakpoint construction shows that if the original automaton is universal then the resulting automaton would be deterministic. That is, starting with a UBW, the breakpoint construction would provide an equivalent DBW. Thus, B' is a DBW and can be dualized in $O(1)$ to a DCW A' for the original language L of $\mathcal{A}$.

Why, for God's sake, $\frac{2}{3}3^n$? Given an NCW $\mathcal{A}$ with a set Q of n states and acceptance set α , the breakpoint construction generates an equivalent DCW $\mathcal{A}'$, in which every state is a pair $\langle S, O \rangle$, where $S \subseteq Q$ and $O \subseteq S \cap (Q \setminus \alpha)$. The state space of $\mathcal{A}'$ is obviously contained in $2^Q \times 2^Q$, which is of size $2^n \times 2^n = 4^n$. Moreover, the state space of $\mathcal{A}'$ is contained in 3^Q by the following observation. Consider a state $\langle S, O \rangle$ of $\mathcal{A}'$. For every state q of $\mathcal{A}$ there are exactly three possibilities: i) $q \in O$; ii) $q \in S \setminus O$; and iii) $q \in Q \setminus S$. Thus, there are up to 3^Q states in $\mathcal{A}'$. Furthermore, for a state $q \in \alpha$ there are only two possibilities (ii and iii), as it cannot be in O . Thus, the state space of $\mathcal{A}'$ would be of size $2^{|\alpha|} \times 3^{|Q \setminus \alpha|}$. Hence, the smaller α is, the larger $\mathcal{A}'$ gets. However, an NCW with an empty α is actually a simple looping automaton (a Büchi automaton whole of whose states are accepting), which can be determinized with only a 2^n state blowup. Hence, the worst case is an NCW with a single accepting state, determinized into a DCW with $2 \times 3^{n-1} = \frac{2}{3}3^n$ states.

Lower bound details

A $\frac{1}{6}3^n$ lower bound is provided in [BKR10] for the translation of a universal Büchi automaton (UBW) to an equivalent NBW. Since UBWs and NCWs are dual and so are NBWs and UCWs, we get from [BKR10] an $\Omega(3^n)$ lower bound for translating an NCW to an equivalent UCW. Since DCW is a special case of UCW, we get an $\Omega(3^n)$ lower bound for determinizing an NCW.

In [BKR10], a more general proof was provided, considering alternating Büchi automata (ABW). In the special case that the source automaton is a UCW (or, equivalently, its dual is an NCW), an exact $\frac{2}{3}3^n$ lower bound can be provided.

Since DPW and DSW are DCW-type [KPB94, KMM04], the lower bound stands for them as well.

6.6 NCW $\rightarrow$ DRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [MH84]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states and index in $\Theta(1)$

Upper bound: $\frac{2}{3}3^n$ [MH84]

Lower bound: 2^n as any determinization

Construction details

By translating the NCW to a DCW (see Section 6.5).

Open

Notice that the tight lower bound of translating an NCW to a DCW does not hold for

translating to a DRW, since the latter is not DCW-type.

6.7 NBW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

State blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states

Upper bound: $\frac{2}{3}3^n$ [BK12]

Lower bound: 2^n [BK12]

Construction details

The construction is the same as in the more general case of translating a Street automaton to a DCW (see Section 6.13).

6.8 NBW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

State blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states [Mic88, Saf89, KPB94, Löd99, Sch09, CZ09]

Construction details

One can translate the NBW to a DRW (see Section 6.10). If a DRW is DBW-recognizable, then it is DBW-type, namely it has an equivalent DBW on the same structure [KPB94].

Lower bound details

The languages in the lower bound of [Mic88] and [Löd99], for the translation of NBW to DRW, are DBW-recognizable. Hence, this lower bound stands for the translation of an NBW to a DBW.

6.9 NBW $\rightarrow$ DPW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$ [Mic88, Saf89, Löd99, Sch09, LW09, CZ09]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(n)$ [Mic88, Löd99, Pit07, Sch09, LW09, SV14]

Best upper bound: $O(n!^2) \approx O((0.13n^2)^n), 2n$ [Pit07, LW09, Sch09]

Best lower bound: $\Omega((1.64n)^n)$ with a large alphabet [CZ09, SV14]

Construction details

The construction due to [Pit07] generates a DPW with $2^{O(n \log n)}$ states and index $2n$. It improves Safra's construction [Saf89] for the NBW $\rightarrow$ DRW translation (see Section 6.10), by replacing the constant node-names with dynamic names. This allows to simulate

the *index appearance record* construction within the deterministic automaton, yielding a parity condition and fewer states.

In [Pit07], the construction upper bound is analyzed to be n^{2n+2} . Yet, a tighter analysis of Piterman's construction, given independently in [LW09] and in [Sch09], shows that the upper bound is only $O(n!^2)$, which is about $O((0.13n^2)^n)$.

In [Sch09], a somewhat different approach is taken, constructing first a deterministic Rabin automaton with acceptance condition on the transitions. The motivation is to separate the structure of the deterministic automaton from the acceptance condition. This automaton is then translated to a DPW, which turns up to be identical to the generated DPW in Piterman's construction.

Lower bound details

Since DPW is a special case of a DRW, the lower bound follows from the translation of NBW to DRW (see Section 6.10). An exact lower bound is provided in [SV14].

6.10 NBW $\rightarrow$ DRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$ [Mic88, Saf89, Löd99, Sch09, CZ09]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(n)$ [Mic88, Saf89, Löd99, Sch09, CZ09]

Exact bounds:

Best upper bound with a small index: $O(n!^2) \approx O((0.13n^2)^n)$, $2n$ [Pit07, LW09, Sch09]

Best upper bound for an alphabet Σ that is small: $|\Sigma|O((1.65n)^n)$, $2^n - 1$ [Sch09]

Best upper bound for a large alphabet: $O((2.66n)^n)$, $2^n - 1$ [Sch09]

Best lower bound for a large alphabet: $\Omega((1.64n)^n)$ [CZ09]

Construction details

The construction of [Saf89] is known as the *Safra-construction*, using *history trees*. A simple explanation of the construction can be found at <http://www.cs.cornell.edu/courses/cs686/2003sp/Handouts/safra.pdf>.

In [Pit07], Safra's construction is improved, providing fewer states, and generating a DPW rather than a DRW (see Section 6.9).

In [Sch09], a somewhat different approach is taken, constructing first a deterministic Rabin automaton with acceptance condition on the transitions. The motivation is to separate the structure of the deterministic automaton from the acceptance condition. This automaton is then translated to a regular DRW. Note that in Schewe's construction the index is $2^n - 1$, compared to an $O(n)$ index in Safra's and Piterman's constructions. This is due to a different naming mechanism for the nodes in the history trees. Schewe's construction does not give explicit names to the nodes, as opposed to the traditional Safra-construction.

Lower bound details

The lower bound of [Mic88] shows that there is no equivalent DRW with less than $2^{\Omega(n \log n)}$ states and index in $O(n)$. The lower bound of [Löd99] extends the result,

showing that there is no equivalent DRW with less than $2^{\Omega(n \log n)}$ states regardless of the index. An exact lower bound, for a large alphabet, is provided in [CZ09].

6.11 NBW $\rightarrow$ DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$ [Saf89, Löd99, Pit07, SV14]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(n)$ [Saf89, Löd99, Pit07, SV14]

Construction details

The translation of [Saf89] goes through a DRW (see Section 6.10), which is then translated to a DSW (see Section 5.8). The former yields a DRW with $2^{O(n \log n)}$ states and index $k = n$ [Saf89]. The state blowup of DRW $\rightarrow$ DSW is only exponential in the index of the original DRW, thus leaving the $2^{O(n \log n)}$ state blowup [Saf89].

In [Pit07], Safra’s construction is improved, providing fewer states, and generating a DPW (and thus also a DSW) rather than a DRW (see Section 6.9).

Lower bound details

An asymptotic lower bound is provided in [Löd99], and then sharpened in [SV14].

6.12 NBW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{2^{O(n \log n)}}$ [Saf89]

Lower bound: $2^{2^{\Omega(n)}}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states results in an automaton with:

Upper bound: $2^{O(n \log n)}$ states and index in $2^{2^{O(n \log n)}}$ [Saf89]

Lower bound: $2^{\Omega(n \log n)}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, Yan08, Bok17a]

Construction details

One can translate first the NBW to a DRW (see Section 6.10), getting $2^{O(n \log n)}$ states and an index in $O(n)$ [Saf89]. The next translation of a DRW to a DMW (see Section 5.2) keeps the DRW’s structure and only enlarges the index. The maximal Muller index may be exponential in its number of states, listing all possible subsets.

Lower bound details

The inevitable state blowup is provided in [Yan08]; As a first step, the family of NBWs that [Yan08] uses for the lower bound proof consists of “full automata”, namely automata that operate over a very rich alphabet, and in the second step a corresponding family of NBWs over a fixed alphabet is provided. The inevitable index blowup is provided in [Saf89, Bok17a] (see Section 6.4).

6.13 NPW, NRW, and NSW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\frac{2}{3}3^n$ [BK12]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $\frac{2}{3}3^n$ states [BK12]

Construction details

Given an NSW $\mathcal{A}$ that is NCW-recognizable, one can translate it to an equivalent NCW $\mathcal{B}$ over the *augmented subset construction* of $\mathcal{A}$ [BK12]. The latter is the product of $\mathcal{A}$ with its subset construction. [BK12]. For a Rabin automaton $\mathcal{A}$ with index k , one can build an equivalent NCW $\mathcal{B}$, when possible, over k copies of the augmented subset construction of $\mathcal{A}$ [BK12].

The key to the translation to a DCW is the observation that the augmented subset construction is transparent to additional applications of the subset construction. Indeed, applying the subset construction on $\mathcal{B}$ with state space $Q \times 2^Q$, one ends up in a deterministic automaton with state space $\{\langle E, E \rangle : E \in 2^Q\}$, which is clearly isomorphic to 2^A . This transparency also applies to a union of some identical copies of the augmented subset construction [BK12].

Now, an NCW can be translated to an equivalent DCW via the breakpoint construction [MH84] (see Section 6.5). The space state of the generated DCW is completely based on the standard subset construction, in the sense that two different automata, $\mathcal{C}$ and $\mathcal{C}'$, having the same subset construction will have the same state space in their breakpoint construction. Hence, an NCW $\mathcal{B}$ generated by the augmented subset construction from an automaton $\mathcal{A}$ with n states will have no more than $\frac{2}{3}3^n$ states.

Lower bound details

Since all these automata classes are also NCW-type, the lower bound for determinizing an NCW is valid (see Section 6.5).

6.14 NPW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$ [Saf92, Pit07, CZ12, SV14]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states [Saf92, Pit07, CZ12, SV14]

Construction details

One can translate the original automaton to a DRW (see Section 6.15). If the constructed DRW is DBW-recognizable, then it is DBW-type, namely it has an equivalent DBW on the same structure [KPB94].

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DBW (see Section 6.8).

6.15 NPW $\rightarrow$ DPW, DSW and DRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n \log n)}$ [Saf92, Pit07, CZ12, SV14]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n \log n)}$ states and index in $\Theta(nk)$ [Saf92, Pit07, CZ12, SV14]

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DRW [Mic88] and to a DSW [Löd99]. Tighter lower bounds are provided in [SV14].

6.16 NPW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{2^{O(n \log n)}}$ [Saf89, Pit07, CZ12, SV14]

Lower bound: $2^{2^{\Omega(n)}}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Upper bound: $2^{O(n \log n)}$ states and index in $2^{2^{\Omega(n \log n)}}$ [Saf89, Pit07]

Lower bound: $2^{\Omega(n \log n)}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

Construction details

One can translate the original automaton to a DRW (see Section 6.15), and then to a DMW over the same state space, and with an index that is up to exponentially larger than the number of states.

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DMW (see Section 6.12).

6.17 NRW $\rightarrow$ DBW

*The state blowup involved in the translation is an **open problem**.*

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n^2 \log n)}$ [Saf92, Pit07]

Lower bound: $2^{\Omega(n \log n)}$ [Mic88, Löd99]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Upper bound: $2^{O(nk \log n)}$ states [Saf92, Pit07]

Lower bound: $2^{\Omega(n \log n)}$ states [Mic88, Löd99]

Construction details

One can translate the original automaton to a DRW (see Section 6.18). If a DRW is DBW-recognizable, then it is DBW-type, namely it has an equivalent DBW on the same structure [KPB94].

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DBW (see Section 6.8).

Open

Regarding the state blowup, it is still open whether the construction requires $nk \log nk$ in the exponent, and how large is the state blowup when the index k is exponential in n . Note that the lower bound of translating an NRW to a DRW (see Section 6.18) does not apply here, since the NRW languages that are used in the lower bound proof are not DBW-recognizable.

6.18 NRW $\rightarrow$ DPW, DRW and DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n^2 \log n)}$ [Saf89, Pit07, CZL09]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $2^{\Theta(nk \log n)}$ states and index in $\Theta(nk)$ [Saf89, Pit07, CZL09]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{2^{\Theta(n)}}$ states and index in $2^{\Theta(n)}$ [Saf89, Pit07, CZL09]

Construction details

[Folklore]: An NRW $\mathcal{A}$ with n states and index k can be viewed as the union of k NRWs, $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k$, each having n states and index 1. Each NRW $\mathcal{A}_i$ can be translated to an NBW $\mathcal{B}_i$ with $O(n)$ states (see Section 7.10). Each NBW $\mathcal{B}_i$ can be translated to a DRW $\mathcal{R}_i$ [Saf89] or to a DPW $\mathcal{P}_i$ [Pit07] with $2^{O(n \log n)}$ states and index n (see Section 6.9). The union of these DPWs, which is equivalent to the original NRW $\mathcal{A}$, is recognized by a DPW $\mathcal{P}$ that can be defined over the product of their states. That is, for every $1 \leq i \leq k$, let Q_i be the states of $\mathcal{P}_i$. Then, the states of $\mathcal{P}$ are $Q_1 \times Q_2 \times \dots \times Q_k$, and its index is kn . Thus, $\mathcal{P}$ has $(2^{O(n \log n)})^k = 2^{O(nk \log n)}$ states.

Lower bound details

By [CZL09], the state blowup in the complementation of an NRW over a fixed alphabet with n states and index k to any automaton of the “common types” is $2^{\Omega(nk \log n)}$. For a big enough k , it is shown to be doubly-exponential. More precisely, for every $\epsilon > 0$ and $k > 2^{n^{1-\epsilon}}$, the lower bound is in $\Omega(2^{(2^{n^{1-\epsilon}})n \log n})$. By “common types”, [CZL09]

refers to any automata class whose acceptance condition is based on the set of states that are visited infinitely often.

Assume, by way of contradiction, a DRW (resp. DSW) equivalent to the original NRW with less states. Then this DRW (resp. DSW) can be dualized over the same structure, to a DSW (resp. DRW), leading to a contradiction.

6.19 NRW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{2^{O(n^2 \log n)}}$ [Folklore]

Lower bound: $2^{2^{\Omega(n)}}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{\Theta(nk \log n)}$ states and index in $2^{2^{\Omega(nk \log n)}}$ [Folklore]

Lower bound: $2^{\Theta(nk \log n)}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

For a big $k \in \Omega(2^n)$:

Upper bound: $2^{2^{\Theta(n)}}$ states and index in $2^{2^{O(nk \log n)}}$ [Folklore]

Lower bound: $2^{2^{\Theta(n)}}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

Construction details

One can translate the NRW to a DRW (see Section 6.18), getting $2^{O(nk \log n)}$ states, and then translate the DRW to a DMW over the same state space and with an index that is up to exponentially larger than the number of states.

Lower bound details

Size lower bound. Corresponds to the index blowup.

State lower bound. As in the translation of an NRW to a DPW (see Section 6.18).

Index lower bound. As in the translation of an NRW to a DMW (see Section 6.4).

6.20 NSW $\rightarrow$ DBW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n^2 \log n)}$ [CZ12]

Lower bound: $2^{\Omega(n \log n)}$ [Mic88, Löd99]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(n \log n + nk \log k)}$ states [Saf89, Pit07, CZ12]

Lower bound: $2^{\Omega(n \log n)}$ [Mic88, Löd99]

For a big $k \in \Omega(2^n)$:

Upper bound: $2^{O(n^2 \log n)}$ states [CZ12]

Lower bound: $2^{\Omega(n \log n)}$ states [Mic88, Löd99]

Construction details

One can translate the original automaton to a DRW (see Section 6.23). If a DRW is DBW-recognizable, then it is DBW-type, namely it has an equivalent DBW on the same structure [KPB94].

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DBW (see Section 6.8).

6.21 NSW $\rightarrow$ DPW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n^2 \log n)}$ [Saf92, Pit07, CZ11a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $2^{\Theta(nk \log nk)}$ states and index in $\Theta(nk)$ [Saf92, Pit07, CZ11a]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n^2 \log n)}$ states and index in $\Theta(n^2)$ [CZ12, CZ11a]

Construction details

The translation due to [Saf92], starting with an NSW with n states and index k , generates a DRW with $(12)^{n(k+1)} n^{n(k+2)} (k+1)^{2n(k+1)}$ states and index $n(k+1)$. Due to the small generated index, the DRW can be translated to a DPW with asymptotically the same number of states (see Section 5.8).

In [Pit07], Safra's construction is improved, providing fewer states, and generating a DPW rather than a DRW. The generated DPW has $n^{n(k+2)+2} (k+1)^{2n(k+1)}$ states and index $2n(k+1)$.

For a big k , the construction in [CZ12, Theorem 2] generates a DRW with $2^{O(n^2 \log n)}$ states and $O(n^2)$ Rabin pairs, which can be translated to a DPW with $2^{O(n^2 \log n)}$ states and $O(n^2)$ priorities (see Section 5.8).

Lower bound details

As the lower bound in the translation to DSW (see Section 6.22), just that for parity automata, a lower bound on the size also implies a lower bound on the number of states.

6.22 NSW $\rightarrow$ DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n^2 \log n)}$ [Pit07, CZ11a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(nk \log nk)}$ states and index in $\Theta(nk)$ [Saf92, Pit07, CZ11a]

Lower bound: $2^{\Omega(n \log n)}$ states [Mic88, Löd99]

For a big $k \in \Omega(2^n)$:

Upper bound: $2^{O(n^2 \log n)}$ states and index in $\Theta(n^2)$ [CZ12, CZ11a]

Lower bound: $2^{\Omega(n \log n)}$ states [Mic88, Löd99]

Construction details

As in the translation to DPW (see Section 6.21).

Lower bound details

Size lower bound. By [CZ11a], the complementation of an NSW $\mathcal{A}$ into an NSW $\overline{\mathcal{A}}$ involves a size blowup of $2^{\Omega(n^2 \log n)}$. Let $\mathcal{B}$ be a DSW equivalent to $\mathcal{A}$. Then, the dual of $\mathcal{B}$ is a DRW $\overline{\mathcal{B}}$ (of the same size as $\mathcal{A}$) equivalent to $\overline{\mathcal{A}}$. Now, every DRW can be translated to an NSW with up to a quadratic size blowup (see Section 7.11). Hence, the size of $\mathcal{B}$ is in $(2^{\Omega(n^2 \log n)})^2 = 2^{\Omega(n^2 \log n)}$. Note, however, that it does not imply a lower bound on the number of states—It might be that the translation generates a big index, which will dominate the size, over a smaller number of states.

State lower bound. The lower bound follows from the lower bound in the translation of an NBW to a DSW [Löd99] (see Section 6.11).

6.23 NSW $\rightarrow$ DRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n^2 \log n)}$ [CZ11a, CZ12]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $2^{\Theta(n \log n + nk \log k)}$ states and index in $\Theta(nk)$ [Saf92, Pit07, CZ11a, CZ12]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n^2 \log n)}$ states and index in $\Theta(n^2)$ [Saf92, Pit07, CZ11a, CZ12]

Construction details

The translation due to [Saf92], starting with an NSW with n states and index k , generates a DRW with $(12)^{n(k+1)} n^{n(k+2)} (k+1)^{2n(k+1)}$ states and index $n(k+1)$.

A different determinization approach, based on alternation-removal in tree-automata, is given in [MS95]. It results with a DRW of the same state space as with Safra's construction.

In [Pit07], Safra's construction is improved, providing fewer states, and generating a DPW rather than a DRW. The generated DPW has $n^{n(k+2)+2} (k+1)^{2n(k+1)}$ states and index $2n(k+1)$.

In [CZ12], the tight blowup with respect to $2^{\Theta(\cdot)}$ was achieved.

Lower bound details

In [CZ11a], the aforementioned lower bounds were established for the complementation of an NSW $\mathcal{A}$ into an NSW $\overline{\mathcal{A}}$. Assume toward contradiction an NRW $\mathcal{B}$ smaller than $\overline{\mathcal{A}}$. Then the dual of $\mathcal{B}$, denoted by $\overline{\mathcal{B}}$, is a DSW equivalent to $\overline{\mathcal{A}}$, but smaller than it, contradicting the complementation lower bound.

6.24 NSW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n^2 \log n)}$ [Saf89, Pit07, CZ12]

Lower bound: $2^{2^{\Omega(n)}}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(n \log n + nk \log k)}$ states and index in $2^{2^{\Omega(n \log n + nk \log k)}}$ [Saf89, Pit07, CZ12]

Lower bound: $2^{\Omega(n \log n)}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

For a big $k \in \Omega(2^n)$:

Upper bound: $2^{O(n^2 \log n)}$ states and index in $2^{2^{\Omega(n^2 \log n)}}$ [Saf89, Pit07, CZ12]

Lower bound: $2^{O(n \log n)}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

Construction details

One can translate the original automaton to a DRW (see Section 6.23), and then to a DMW over the same state space, and with an index that is up to exponentially larger than the number of states.

Lower bound details

The lower bound follows from the lower bound in the translation of an NBW to a DMW (see Section 6.12).

6.25 NMW $\rightarrow$ DCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $O(3^n)$ [BK12]

Lower bound: $\Omega(2^n)$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a big $k \in \Omega(2^n)$:

Tight bound: $\Theta(3^n)$ states [BK12]

Construction details

For an NMW $\mathcal{A}$ with index k , one can build an equivalent NCW $\mathcal{B}$, when possible, over

k copies of the augmented subset construction of $\mathcal{A}$ [BK12]. The latter is the product of $\mathcal{A}$ with its subset construction.

The key to the translation to a DCW is the observation that the augmented subset construction is transparent to additional applications of the subset construction. Indeed, applying the subset construction on $\mathcal{B}$ with state space $Q \times 2^Q$, one ends up in a deterministic automaton with state space $\{\langle E, E \rangle : E \in 2^Q\}$, which is clearly isomorphic to 2^A . This transparency also applies to a union of some identical copies of the augmented subset construction [BK12].

Now, an NCW can be translated to an equivalent DCW via the breakpoint construction [MH84] (see Section 6.5). The space state of the generated DCW is completely based on the standard subset construction, in the sense that two different automata, $\mathcal{C}$ and $\mathcal{C}'$, having the same subset construction will have the same state space in their breakpoint construction. Hence, an NCW $\mathcal{B}$ generated by the augmented subset construction from an automaton $\mathcal{A}$ with n states will have no more than $\frac{2}{3}3^n$ states.

Lower bound details

An $\Omega(3^n)$ state and size lower bound is provided for the determinization of an NCW (see Section 6.5). There exists an equivalent NMW over the same state space of the NCW, implying a tight blowup result with respect to the number of states. Yet, the index of this NMW is exponential in its number of states, for which the this lower bound does not stand for the size blowup.

An $\Omega(2^n)$ lower bound for the determinization of all automata types is trivial, by reduction to the case of finite words.

6.26 NMW $\rightarrow$ DBW

*The state blowup involved in the translation is an **open problem**.*

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n^3 \log n)}$ [Folklore]

Lower bound: $2^{\Omega(n)}$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Upper bound: $2^{O(n^2 k \log nk)}$ states [Folklore]

Lower bound: $2^{\Omega(n \log n)}$ states [Mic88]

Construction details

One can translate the original automaton to a DRW (see Section 6.23). If a DRW is DBW-recognizable, then it is DBW-type, namely it has an equivalent DBW on the same structure [KPB94].

Lower bound details

Size lower bound. The $2^{O(n \log n)}$ state lower bound in the determinization of NBWs [Mic88] applies to the state blowup in the determinization of Muller automata, as every NBW has an equivalent NMW over the same state space. However, these NMWs have index that is exponential in the number of their states, for which reason the lower bound does not apply for the size blowup.

State lower bound. The lower bound follows from the lower bound in the translation of an NBW to a DBW (see Section 6.8).

Open

Considering the state blowup, it is still open whether the construction requires $n^2 k \log nk$ in the exponent, and how large is the state blowup when the index k is exponential in n . Note that the lower bound of translating an NRW to a DRW (see Section 6.18) does not apply here, since the chosen NRW languages are not DBW-recognizable.

6.27 NMW $\rightarrow$ DPW, DRW and DSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{O(n^3 \log n)}$ [Folklore]

Lower bound: $2^{\Omega(n)}$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(n^2 k \log nk)}$ states and index in $O(n^2 k)$ [Folklore]

Lower bound: $2^{\Omega(nk \log n)}$ states [CZL09]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{2^{\Theta(n)}}$ states [CZL09]

Construction details

One can translate first the NMW to an NBW (see Section 7.17), getting $O(n^2 k)$ states. Next, the NBW can be translated to a DPW with $2^{O(n^2 k \log nk)}$ states and index $O(n^2 k)$.

Lower bound details

Size lower bound. The state lower bound in the determinization of NRWs [CZL09] applies to the state blowup in the determinization of Muller automata, as every NRW has an equivalent NMW over the same state space. However, these NMWs have index that is exponential in the number of their states, for which reason the lower bound does not apply for the size blowup.

State lower bound. Follows from the lower bound in the translation of an NRW to a DRW (see Section 6.18).

6.28 NMW $\rightarrow$ DMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $2^{2^{O(n^3 \log n)}}$ [Folklore]

Lower bound: $2^{2^{\Omega(n)}}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $2^{O(n^2 k \log nk)}$ states and index in $2^{2^{O(n^2 k \log nk)}}$

Lower bound: $2^{\Omega(nk \log k)}$ states and index in $2^{2^{\Omega(n)}}$

For a big $k \in \Omega(2^n)$:

Upper bound: $2^{2^{\Theta(n)}}$ states and index in $2^{2^{O(nk \log n)}}$ [Folklore]

Lower bound: $2^{2^{\Theta(n)}}$ states and index in $2^{2^{\Omega(n)}}$ [Saf89, CZL09, Bok17a]

Construction details

One can translate the NMW to a DRW (see Section 6.27), getting $2^{O(n^2 k \log nk)}$ states, and then translate the DRW to a DMW over the same state space and with an index that is up to exponentially larger than the number of states.

Lower bound details

Size lower bound. Corresponds to the index blowup.

State lower bound. As in the translation of an NRW to a DMW (see Section 6.19).

Index lower bound. As in the translation of an NWW to a DMW (see Section 6.4), while changing a little the considered languages, such that an NMW can express them using a small index.

7 Deterministic and nondeterministic to Nondeterministic

7.1 All $\rightarrow$ NMW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [Saf89, Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: n states and index in $2^{\Theta(n)}$ [Saf89, Bok17a]

Construction details

All the considered automata classes have equivalent Muller automata on the same state space. Yet, the Muller index might be exponential in the number of states.

Lower bound details

The high descriptive level of the Muller condition has its cost in the index. In [Saf89], Safra shows a family of languages L_n that can be recognized by NBWs with n states, whereas an equivalent DMW must have an index in $\Omega(2^{2^n})$. Safra's languages, however, use an alphabet that is exponential in the number of states. Hence, size wise, the result does not provide an exponential lower bound.

In [Bok17a], Safra's result is strengthened, considering a family L_n of deterministic looping automata (Büchi automata whole of whose states are accepting) over a fixed alphabet, and showing that an equivalent DMW must have an index in $\Omega(2^n)$. Thus, we get the exponential size blowup from all the considered automata classes.

7.2 DCW and NCW $\rightarrow$ NWW and NBW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n)$ [Kur87, AKL08]

State blowup

The translation of an automaton with n states results in an automaton with:

Tight bound: $2n - 1$ states

Upper bound: [Kur87]

Lower bound: [AKL08]

Since there is the same tight bound for translating to both NWW and NBW, and every NWW is also an NBW, it is enough to show the upper bound with respect to NWW and the lower bound with respect to NBW. Furthermore, the lower bound due to [AKL08] starts with a DCW, thus providing a tight bound also for the DCW $\rightarrow$ NWW and NBW translation.

Construction details

Given an NCW $\mathcal{A}$ with n states and m transitions, in which α are the accepting states (should be visited only finitely often), one can build an equivalent NWW $\mathcal{A}'$ with $2n - |\alpha|$ states and $3m$ transitions. The NWW $\mathcal{A}'$ is built by taking two copies of $\mathcal{A}$, deleting all the states in α from the second copy, and making only the remaining states of the second copy accepting. Transitions are also added to enable the automaton to move from the first copy to the second copy, but not back. The idea is that since an accepting run of $\mathcal{A}$ visits states in α only finitely many times, it can be simulated by a run of $\mathcal{A}'$ that switches to the second copy when states in α are no longer needed.

7.3 DBW $\rightarrow$ NWW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [BJW01, KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

Construction details

NWW, NCW and DCW have the same expressive power. Thus, if a DBW can be translated to an NWW it can also be translated to a DCW. Furthermore, since $DBW \cap DCW = DWW$ [BJW01], it follows that the original DBW is DWW-recognizable. Hence, by the DWW-typeness of DBW [KMM04], DBW are also NWW-type (getting a DWW).

7.4 DBW $\rightarrow$ NCW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KMM04].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

Construction details

NCWs and DCWs have the same expressive power. Thus, if a DBW can be translated to an NCW it can also be translated to a DCW. Since DBW are DCW-type [KMM04], it follows that if a DBW can be translated to an NCW it can be done over the original structure of the DBW (ending with a DCW).

7.5 NBW, NPW, NRW, NSW, and NMW $\rightarrow$ NWW and NCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [BK12]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states [BK12]

Construction details

NWW is a special case of NCW, and every NCW can be translated to an NWW with only a linear state blowup (see Section 7.2). Hence, the translations to NWW and to NCW have the same state blowup.

Note that the tightness is in the exponent but not in the base, meaning that it is not $\Theta(2^n)$. The following table details the known upper bounds for the translations [BK12].

	Small k	Large k
NBW	$n2^n$	$n2^n$
NPW	$n2^n$	$n2^n$
NRW	$kn2^n$	$\frac{2}{3}3^n$
NSW	$n2^n$	$n2^n$
NMW	$kn2^n$	$\frac{2}{3}3^n$

Note that for Streett (and parity) the state blowup is only $n2^n$, regardless of the index k . The 3^n state blowup for Rabin and Muller automata with a large index is achieved by translating them to a DCW (see Section 6.13).

The translation is based on the *augmented subset construction* [BK12], which is the product of the original automaton with its subset construction. Given an NSW $\mathcal{A}$ that is NCW-recognizable, one can translate it to an equivalent NCW over the augmented subset construction of $\mathcal{A}$. The latter is not enough for translating a Rabin or Muller automaton to an NCW [BK12]. Yet, for such an automaton with index k , one can build an equivalent NCW, when possible, with k copies of the augmented subset construction of $\mathcal{A}$ [BK12].

Lower bound details

An $\Omega(2^n)$ lower bound is shown [BK12] for translating an NBW to an NWW, thus valid

also for the translation of an NPW, NRW, and NSW. As for the translation of an NMW, it provides a lower bound on the state blowup, but not necessarily on the size blowup, as the index in the NMW might be exponential in the number of states. Yet, a close look on the Büchi automata used in the lower bound of [BK12] suggests that they can be expressed by NMWs over the same structure and with a constant index of 2.

7.6 DPW and DSW $\rightarrow$ NWW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2n - 1$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2n - 1$ states

Construction details

NWW, NCW and DCW have the same expressive power. Thus, if a DPW or DSW can be translated to an NWW it can also be translated to a DCW. Since DPW and DSW are DCW-type [KPB94, KMM04], it follows that the state blowup is at most as in translating a DCW to an NWW. The latter is $2n - 1$ [Kur87] (see Section 7.2).

Lower bound details

DCW can be viewed as a special case of DPW and DSW. Thus, the $2n - 1$ lower bound for translating a DCW to an NWW [AKL08] follows to the current case.

7.7 DPW and DSW $\rightarrow$ NCW

A translation is **not always possible** (see Section 1.4).

There is **no blowup** in the translation.

The new automaton is *typed* to the original automaton [KPB94].

That is, one can translate the original automaton to the target automaton by only changing the acceptance condition, while keeping the structure of the original automaton. Furthermore, the size of the new acceptance condition is not larger than that of the original one (see Section 1.3).

Construction details

The parity condition can be viewed as a Streett condition (see Section 1.3). By [KPB94], DRW is DBW-type, and by dualization we have that DSW are DCW type.

NCWs and DCWs have the same expressive power. Thus, if a DSW can be translated to an NCW it can also be translated to a DCW. Since DSW are DCW-type, it follows that if a DSW can be translated to an NCW it can be done over the original structure of the DSW (ending with a DCW).

7.8 DPW, NPW $\rightarrow$ NBW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Cho74, SN99]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $\Theta(kn)$ states [Cho74, SN99]

Construction details

An NPW with index k can be viewed as an NRW with index k (see Section 1.3). The latter can be translated to an NBW with $2kn$ states (see Section 7.10). Since the index of a DPW can be at most n , the worst case is an equivalent NBW with $2n^2$ states.

Lower bound details

In [SN99, p. 15], there is the family of languages $L_{n,k}$, as detailed below, for every $n, k > 1$, over the alphabet (digits) $\Sigma = \{0, 1, 2, \dots, k\}$, for which there is a DPW with $O(n)$ states and index k , while it is shown that equivalent NBWs must have $\Omega(nk)$ states. The language $L_{n,k} = \{w \in (0^n(1 + 2 + 3 + \dots + k))^\omega \mid \text{the maximal digit that appears infinitely often is even}\}$ consists of words in which the maximal digit that appears infinitely often is even, and the digit 0 only appears in blocks of n consecutive 0's.

The lower-bound languages of [SN99] use an alphabet that is linear in the number of states of the DPW, yet they can be changed to use a fixed alphabet, by a technique analogous to the one that Löding used for enhancing Michel's lower bound (for the NBW $\rightarrow$ DBW translation) to use a fixed alphabet rather than a linear one.

7.9 DRW $\rightarrow$ NWW and NCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $O(n^2)$ [Bok17b]

Lower bound: $\Omega(n)$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

A DRW with n states and index k can be translated to a DCW (see Section 5.6) with $O(nk)$ states [Bok17b], which has the same expressive power as an NCW, and can be translated to an NWW with only a linear state blowup (see Section 7.2).

Lower bound details

Size lower bound. There is no non-trivial bound.

State lower bound. It is shown in [Bok17b] that for every $n \in \mathbb{N}$, there is an NCW-recognizable DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NCWs have at least 2^n states.

7.10 DRW and NRW $\rightarrow$ NBW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Cho74, SN99]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $O(kn)$ states [Cho74]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states [Cho74, Bok17b]

Construction details

An NRW can be translated to an NBW as follows.

An NRW[1], which is an NRW with a single accepting pair, with n states and m transitions can be translated to an NBW with $2n - 1$ states and $3m$ transitions. Indeed, consider an NRW[1] with a set of states α that should be visited infinitely often and a set of states β that should be visited only finitely often. The construction of an equivalent NBW is quite similar to translating an NCW to an NBW (see Section 7.2): There are two copies of the original NRW, while the second copy does not have the β states. The first copy “guesses” when the states in β can be avoided and jumps to the second copy. The second copy cannot return to the first copy (thus avoids the states of β), and has α as its accepting states.

An NRW[k] can be viewed as a union of k NRW[1]’s. Thus, we can use the nondeterminism of the required NBW to have k different NBWs, each equivalent to an NRW[1]. Hence, we get an NBW with $O(kn)$ states and $O(km)$ transitions.

Lower bound details

Size lower bound. Follows from the lower bound in DPW $\rightarrow$ NBW (see Section 7.8).

State lower bound. Follows from the lower bound in DRW $\rightarrow$ NSW (see Section 7.11).

7.11 DRW $\rightarrow$ NPW and NSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $O(n^2)$ [Cho74]

Lower bound: $\Omega(n)$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $O(kn)$ states and index in $O(1)$ [Cho74]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states and index in $\Theta(1)$ [Cho74, Bok17b]

Construction details

By translating the given DRW as an NRW to NBW (see Section 7.10).

Lower bound details

Size lower bound. There is no non-trivial bound.

State lower bound. In [Bok17b], it is shown that for every $n \in \mathbb{N}$, there is a DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NSWs have at least 2^n states.

7.12 NRW $\rightarrow$ NPW and NSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Cho74, Bok22]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $O(kn)$ states and index in $O(1)$ [Cho74]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states and index in $\Theta(1)$ [Cho74, Bok17b]

Construction details

By translating an NRW to an NBW (see Section 7.10).

Lower bound details

Size lower bound. The lower bound in [Bok22] provides a family $\{\mathcal{A}_n\}_{n \geq 1}$ of NRWs with $8n + 4$ states, $16n$ transitions and $n+1$ acceptance pairs, for which equivalent nondeterministic Streett automata have at least $n^2/2$ states.

State lower bound. In [Bok17b], it is shown that for every $n \in \mathbb{N}$, there is a DRW over a two-letter alphabet with $4n$ states and $6n$ transitions, for which equivalent NSWs have at least 2^n states.

7.13 DSW and NSW $\rightarrow$ NBW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [SV89]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $n2^{\Theta(k)}$ states [SV89]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states [SV89]

Construction details

The construction follows the following idea. Consider an NSW $\mathcal{A}$ with n states and m transitions, where the state space is Q and the acceptance pairs are B_i, G_i , for $1 \leq i \leq k$. By the Streett acceptance condition, every accepting run r either avoids B_i from some

position onwards or visits G_i infinitely often, for every $1 \leq i \leq k$. An equivalent NBW may “guess” for every $1 \leq i \leq k$ whether r avoids B_i or visits G_i infinitely often. There are exactly 2^k such possible guesses. For each such guess, there is an equivalent NBW with $O(nh)$ states and $O(mh)$ transitions, where $h \leq k$ is the number of G_i ’s which should be visited infinitely often. The construction, for each guess, is very similar to the translation of an NMW[1] to an NBW (see Section 7.17). Thus, in the resulting NSW there are up to $nk2^k \in n2^{O(k)}$ states.

For a big k , one can also consider the NSW as an NMW and translate it to an NBW with $2^{O(n)}$ state blowup (see Section 7.17).

Lower bound details

In [Saf89, Lemma 2.3], there is a family of languages L_n , for $n > 0$, over a fixed alphabet, such that L_n is recognized by a DSW with $O(n)$ states, transitions, and index, while an equivalent NBW requires $\Omega(2^n)$ states.

7.14 DSW and NSW $\rightarrow$ NPW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [Cho74, SV89]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Upper bound: $O(n2^k)$ states and index in $O(1)$ [Cho74]

Lower bound: $\Omega(\sqrt{n} 2^{k/2})$ states [SV89]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ [Cho74, SV89]

Construction details

By translating to an NBW (see Section 7.13).

Lower bound details

An NPW with n states and index k can be translated to an NBW with $O(nk)$ states. Moreover, the index k is at most as big as n , implying a quadratic upper bound for the state blowup of NPW $\rightarrow$ NBW (see Section 7.8). Thus, a lower bound for the translation of some automata-class $\mathcal{C}$ to an NBW implies a corresponding square-root lower bound for the translation of $\mathcal{C}$ to an NPW. In this case, the lower bound for the translation to an NBW is $\Omega(n2^k)$ (see Section 7.13).

7.15 DSW and NSW $\rightarrow$ NRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $2^{\Theta(n)}$ [Cho74, SV89]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $\Theta(n^2)$ states and index in $2^{\Theta(n)}$ [Bok17b]

Construction details

By translating the NSW to an NMW (with no state blowup and up to a 2^n index blowup) and then the NMW to an NRW (see Section 7.19).

Lower bound details

Size lower bound. The exponential lower bound on the size blowup involved in the translation of DSWs to NBWs (see Section 7.13) follows to the translation of DSWs to NRWs, as the NRW $\rightarrow$ NBW translation only involves a quadratic size blowup.

State lower bound. In [Bok17b], there is a family $\{\mathcal{D}_n\}_{n \geq 1}$ of DSWs over a two-letter alphabet with $2n+1$ states, $3n$ transitions, and index n , for which equivalent NRWs are shown to have at least $n^2/2$ states.

Index lower bound. Consider the DSWs that are used for the exponential lower bound on the size blowup. When translated to NRWs as per the construction in [Bok17b], the constructed automata have only $O(n^2)$ states and transitions, but an exponential size, which is due to an exponential index.

7.16 DMW $\rightarrow$ NWW and NCW

A translation is **not always possible** (see Section 1.4).

Size blowup

The translation of an automaton of size n results in an automaton of size:

Upper bound: $O(n^2)$ [Bok17b]

Lower bound: $\Omega(n)$

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $2^{\Theta(n)}$ states [BK12, Bok17b]

Construction details

A DMW with n states and index k can be translated to a DCW (see Section 5.11) with $O(nk)$ states [Bok17b], which has the same expressive power as an NCW, and can be translated to an NWW with only a linear state blowup (see Section 7.2).

Lower bound details

Size lower bound. The DMW that recognizes the language L_n , as described above for the state blowup, has an index exponential in the number of states. Thus, size wise, there is no exponential blowup in the translation to an equivalent NWW.

State lower bound. Follows from the DRW $\rightarrow$ NWW state lower bound (see Section 7.9).

Open

There is currently no lower bound for the size blowup, and a gap between the $2^{O(n)}$ upper bound and the $2^{\Omega(\sqrt{n} \log \sqrt{n})}$ lower bound for the state blowup.

7.17 DMW and NMW $\rightarrow$ NBW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^3)$ [Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $\Theta(n^2k)$ states [Bok17a]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states [SV89]

Construction details

[Folklore]: An NMW with n states and index k can be viewed as a union of k NMWs, each with n states (the same structure) and a single acceptance set. We will show that the latter can be translated to an NBW with n^2 states, thus getting a unified NBW with kn^2 states.

Consider an NMW $\mathcal{A}$ with states Q and a single acceptance set α . We show below, first informally and then formally, how to construct an equivalent NBW $\mathcal{B}$.

The NBW $\mathcal{B}$ has two components. The first component is a full copy of $\mathcal{A}$. The second component has $|\alpha|$ copies of $\mathcal{A}$, each restricted to only contain the states of α . The state q_1 of the first copy of the second component is accepting, and all other states are not. The transitions in the first component are as in $\mathcal{A}$, and in addition allow to move to the corresponding states in the first copy of the second component. In the second component, the transitions are as in $\mathcal{A}$, except that the transitions of q_i in the i th copy move to the next copy modulo $|\alpha|$.

Formally, let $\mathcal{A} = \langle \Sigma, Q, q_0, \delta, \alpha = \{q_1, \dots, q_{|\alpha|}\} \rangle$. We define the NBW $\mathcal{B} = \langle \Sigma, Q', q'_0, \delta', \alpha' \rangle$, where:

- $Q' = Q \times \{0\} \cup \alpha \times \{1, 2, \dots, |\alpha|\}$
- $q'_0 = \langle q_0, 0 \rangle$
- For every $\sigma \in \Sigma$ and $q \in Q$: $\delta'(\langle q, 0 \rangle, \sigma) = \delta(q, \sigma) \times \{0\} \cup (\delta(q, \sigma) \cap \alpha) \times \{1\}$. For every $\sigma \in \Sigma$ and $i \in [1..|\alpha|]$: $\delta'(\langle q_i, i \rangle, \sigma) = \delta(q_i, \sigma) \times \{i+1 \bmod |\alpha|\}$. For every $\sigma \in \Sigma$ and $i \neq j \in [1..|\alpha|]$: $\delta'(\langle q_i, j \rangle, \sigma) = \delta(q_i, \sigma) \times \{j\}$.
- $\alpha' = \langle q_1, 1 \rangle$

Lower bound details

Size lower bound. As in the translation to an NPW (see Section 7.19).

State lower bound. In [SV89], there is an exponential state blowup for translating a DSW with a big k to an NBW. Since every DSW has an equivalent DMW on the same state space, we get the blowup also when translating a DMW.

7.18 DMW and NMW $\rightarrow$ NPW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^3)$ [Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $\Theta(n^2k)$ states and index in $\Theta(1)$ [Bok17a]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states and index in $\Theta(1)$ [SV89]

Construction details

By translating to an NBW (see Section 7.17).

Lower bound details

Size lower bound. The lower bound proof in [Bok17a] builds on the observation that parity automata are a special case of both Rabin and Streett automata. It combines the techniques of the two corresponding lower bounds, from NMW to NRW and to NSW, which are both $\Omega(n^2)$, into an $\Omega(n^3)$ lower bound.

State lower bound. An NPW with n states and index k can be translated to an NBW with $O(nk)$ states. Moreover, the index k is at most as big as n , implying a quadratic upper bound for the state blowup of $\text{NPW} \rightarrow \text{NBW}$ (see Section 7.8). Thus, a lower bound for the translation of some automata-class $\mathcal{C}$ to an NBW implies a corresponding square-root lower bound for the translation of $\mathcal{C}$ to an NPW.

7.19 DMW and NMW $\rightarrow$ NRW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Bok17a, Bok17b]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

Tight bound: $\Theta(n^2)$ states and index in $\Theta(n)$ [Bok17a, Bok17b]

Construction details

In [Bok17b] there is an algorithm to translate every NMW with n states, m transitions, and index k , to an equivalent NRW with $2n^2$ states, $3nm$ transitions, and index k . The construction idea is as follows.

Consider an automaton structure A with n states, and a Muller acceptance set S . A run r is accepting according to S if it visits infinitely often all the states in S and only finitely often the states out of S . Let us look first what can be done with a Rabin automaton that is defined over A . We can easily define a Rabin acceptance pair $\langle B, G \rangle$ that partially corresponds to the Muller acceptance set S —We define B to include all the states out of S , ensuring that they are visited only finitely often. The problem is that the set G cannot force visits in all states of S ; It can only force a visit in some states of S .

In order to force a visit in every state of a set $S = \{q_1, q_2, \dots, q_{|S|}\}$, one can take $|S|$ copies of A (which we call “components”), move from the i -th component to the next one (module $|S|$) upon reaching the state q_i , and setting, say, q_1 in the first component to be the only accepting state. This is, for example, the idea in translating a generalized Büchi automaton to a Büchi automaton.

The problem with the above approach is that we need $|S|$ copies of A for every Muller acceptance set S . As there might be 2^n Muller acceptance sets, we get an exponential state blowup. This blowup is indeed inevitable when translating to a Büchi automaton [SV89], yet it is not clear if and how the Rabin acceptance condition can help. We show that it certainly can, allowing all the Muller sets S to be handled over the same set of components.

The construction in [Bok17b] extends the above approach of having copies of A , by adding a “bridge” between each two copies. A bridge is a limited copy of A , in which all states can only move to the next component. Then, for every Muller acceptance set S , we define a Rabin acceptance pair $\langle B, G \rangle$ that forces for every state $q_i \in S$, a visit to the state q_i of the i -th bridge—We add to B all the states of the i -th bridge, except for q_i . In bridges of a component j , such that $q_j \notin S$, the run can visit any state of S , and we choose it to be the minimal one.

Lower bound details

In [Bok17a] there is a family $\{\mathcal{M}_n\}_{n \geq 1}$ of DMWs with $2n+1$ states, $3n$ transitions, and a single accepting set, for which equivalent NRW are shown to have at least $n^2/2$ states.

7.20 DMW and NMW $\rightarrow$ NSW

A translation is **always possible**.

Size blowup

The translation of an automaton of size n results in an automaton of size:

Tight bound: $\Theta(n^2)$ [Bok17a]

State and Index blowup

The translation of an automaton with n states and index k results in an automaton with:

For a small k :

Tight bound: $\Theta(nk)$ states and index in $\Theta(nk)$ [Bok17a]

For a big $k \in \Omega(2^n)$:

Tight bound: $2^{\Theta(n)}$ states and index in $\Theta(1)$ [Bok17b]

Construction details

[Folklore]: An NMW with n states, and index k can be viewed as a union of k NMWs, each with n states (the same structure) and a single acceptance set. We will show that the latter can be translated to an NSW with $2n$ states and index n , thus getting a unified NSW with $2kn$ states and index kn .

Consider an NMW $\mathcal{A}$ with states Q and a single acceptance set α . We show below, first informally and then formally, how to construct an equivalent NSW.

The NSW $\mathcal{B}$ has two components. The first component is a full copy of $\mathcal{A}$. The second component has a restricted copy of $\mathcal{A}$ that only contains the states of α . The transitions in the first component are as in $\mathcal{A}$, and in addition allow to move to the corresponding states in the second component. The Streett acceptance condition will require all the states of the second component to be visited infinitely often.

Formally, let $\mathcal{A} = \langle \Sigma, Q, q_0, \delta, \alpha = \{q_1, \dots, q_{|\alpha|}\} \rangle$. We define the NSW $\mathcal{B} = \langle \Sigma, Q', q'_0, \delta', \alpha' \rangle$, where:

- $Q' = Q \times \{0\} \cup \alpha \times \{1\}$
- $q'_0 = \langle q_0, 0 \rangle$

- For every $\sigma \in \Sigma$ and $q \in Q$: $\delta'(\langle q, 0 \rangle, \sigma) = \delta(q, \sigma) \times \{0\} \cup (\delta(q, \sigma) \cap \alpha) \times \{1\}$. For every $\sigma \in \Sigma$ and $q \in Q$: $\delta'(\langle q, 1 \rangle, \sigma) = \delta(q, \sigma) \times \{1\}$.
- $\alpha' =$ the pair $\langle \emptyset, \{q_1\} \rangle$ and for every $i \in [1..|\alpha|-1]$, the pair $\langle \{q_i\}, \{q_{i+1}\} \rangle$.

Lower bound details

Size lower bound. In [Bok17a], there is a family $\{\mathcal{D}_n\}_{n \geq 1}$ of DMWs with $2n+1$ states, $4n$ transitions, and $n+1$ accepting sets, for which it is shown that equivalent NSWs have at least $n^2/2$ states.

State lower bound. Follows from the lower bound in the translation of a DRW to an NSW (see Section 7.11).

References

- [AKL08] Benjamin Aminof, Orna Kupferman, and Omer Lev. On the relative succinctness of nondeterministic Büchi and co-Büchi word automata. In *Proc. 15th Int. Conf. on Logic for Programming Artificial Intelligence and Reasoning*, volume 5330 of *Lecture Notes in Computer Science*, pages 183–197. Springer, 2008.
- [BJW01] Berdard Boigelot, Sébastien Jodogne, and Pierre Wolper. On the use of weak automata for deciding linear arithmetic with integer and real variables. In *Proc. Int. Joint Conf. on Automated Reasoning*, volume 2083 of *Lecture Notes in Computer Science*, pages 611–625. Springer, 2001.
- [BK12] Udi Boker and Orna Kupferman. Translating to co-Büchi made tight, unified, and useful. *ACM Trans. Comput. Log.*, 13(4):29:1–29:26, 2012.
- [BKR10] Udi Boker, Orna Kupferman, and Adin Rosenberg. Alternation removal in Büchi automata. In *Proc. 37th Int. Colloq. on Automata, Languages, and Programming*, volume 6199, pages 76–87, 2010.
- [Bok17a] Udi Boker. On the (in)succinctness of Muller automata. In *CSL*, pages 12:1–12:16, 2017.
- [Bok17b] Udi Boker. Rabin vs. Streett automata. In *FSTTCS*, pages 17:1–17:15, 2017.
- [Bok18] Udi Boker. Why these automata types? In *Proceedings of LPAR*, pages 143–163, 2018.
- [Bok22] Udi Boker. Nondeterministic Rabin to Streett automata, 2022. Unpublished.
- [Büc62] Julius R. Büchi. On a decision method in restricted second order arithmetic. In *Proc. Int. Congress on Logic, Method, and Philosophy of Science. 1960*, pages 1–12. Stanford University Press, 1962.
- [CBK90] Edmund M. Clarke, I. Anca Browne, and Robert P. Kurshan. A unified approach for showing language containment and equivalence between various types of ω -automata. In *CAAP '90*, pages 103–116, 1990.

- [CES86] Edmund M. Clarke, E. Allen Emerson, and A. Prasad Sistla. Automatic verification of finite-state concurrent systems using temporal logic specifications. *ACM Transactions on Programming Languages and Systems*, 8(2):244–263, 1986.
- [Cho74] Yaacov Choueka. Theories of automata on ω -tapes: A simplified approach. *Journal of Computer and Systems Science*, 8:117–141, 1974.
- [CZ09] Thomas Colcombet and Konard Zdanowski. A tight lower bound for determinization of transition labeled Büchi automata. In *ICALP*, pages 151–162, 2009.
- [CZ11a] Yang Cai and Ting Zhang. A tight lower bound for Streett complementation. In *FSTTCS*, pages 339–350, 2011.
- [CZ11b] Yang Cai and Ting Zhang. Tight upper bounds for Streett and parity complementation. In *CSL*, pages 112–128, 2011.
- [CZ12] Yang Cai and Ting Zhang. Can nondeterminism help complementation? In *GandALF*, pages 57–70, 2012.
- [CZL09] Yang Cai, Ting Zhang, and Haifeng Luo. An improved lower bound for the complementation of Rabin automata. In *LICS*, pages 167–176, 2009.
- [EL87] E. Allen Emerson and Chin-Laung Lei. Modalities for model checking: Branching time logic strikes back. *Science of Computer Programming*, 8:275–306, 1987.
- [GH82] Yuri Gurevich and Leo Harrington. Trees, automata, and games. In *Proc. 14th ACM Symp. on Theory of Computing*, pages 60–65. ACM Press, 1982.
- [KKV01] Valerie King, Orna Kupferman, and Moshe Y. Vardi. On the complexity of parity word automata. In *Proc. 4th Int. Conf. on Foundations of Software Science and Computation Structures*, volume 2030 of *Lecture Notes in Computer Science*, pages 276–286. Springer, 2001.
- [KMM04] Orna Kupferman, Gila Morgenstern, and Aniello Murano. Typeness for ω -regular automata. In *2nd Int. Symp. on Automated Technology for Verification and Analysis*, volume 3299 of *Lecture Notes in Computer Science*, pages 324–338. Springer, 2004.
- [KPB94] Sriram C. Krishnan, Anuj Puri, and Robert K. Brayton. Deterministic ω -automata vis-a-vis deterministic Büchi automata. In *Algorithms and Computations*, volume 834 of *Lecture Notes in Computer Science*, pages 378–386. Springer, 1994.
- [Kur87] Robert P. Kurshan. Complementing deterministic Büchi automata in polynomial time. *Journal of Computer and Systems Science*, 35:59–71, 1987.
- [KV05] Orna Kupferman and Moshe Y. Vardi. Complementation constructions for nondeterministic automata on infinite words. In *Proc. 11th Int. Conf. on Tools and Algorithms for the Construction and Analysis of Systems*, volume 3440 of *Lecture Notes in Computer Science*, pages 206–221. Springer, 2005.

- [Lan69] Lawrence H. Landweber. Decision problems for ω -automata. *Mathematical Systems Theory*, 3:376–384, 1969.
- [Löd99] Christof Löding. Optimal bounds for the transformation of omega-automata. In *Proc. 19th Conf. on Foundations of Software Technology and Theoretical Computer Science*, volume 1738 of *Lecture Notes in Computer Science*, pages 97–109, 1999.
- [LW09] Wanwei Liu and Ji Wang. A tighter analysis of Piterman’s Büchi determinization. *Inf. Process. Lett.*, 109(16):941–945, 2009.
- [LY08] Christof Löding and Haidi Yue. Memory bounds for winning strategies in infinite games, 2008. Unpublished.
- [McN66] Robert McNaughton. Testing and generating infinite sequences by a finite automaton. *Information and Control*, 9:521–530, 1966.
- [MH84] Satoru Miyano and Takeshi Hayashi. Alternating finite automata on ω -words. *Theoretical Computer Science*, 32:321–330, 1984.
- [Mic88] Max Michel. Complementation is more difficult with automata on infinite words. CNET, Paris, 1988.
- [Mos84] Andrzej W. Mostowski. Regular expressions for infinite trees and a standard form of automata. In *Computation Theory*, volume 208 of *Lecture Notes in Computer Science*, pages 157–168. Springer, 1984.
- [MS95] David E. Muller and Paul E. Schupp. Simulating alternating tree automata by nondeterministic automata: New results and new proofs of theorems of Rabin, McNaughton and Safra. *Theoretical Computer Science*, 141:69–107, 1995.
- [MS97] Oded Maler and Ludwig Staiger. On syntactic congruences for ω -languages. *Theoretical Computer Science*, 183(1):93–112, 1997.
- [MSS86] David E. Muller, Ahmed Saoudi, and Paul E. Schupp. Alternating automata, the weak monadic theory of the tree and its complexity. In *Proc. 13th Int. Colloq. on Automata, Languages, and Programming*, volume 226 of *Lecture Notes in Computer Science*, pages 275 – 283. Springer, 1986.
- [Mul63] David E. Muller. Infinite sequences and finite machines. In *Proc. 4th IEEE Symp. on Switching Circuit Theory and Logical design*, pages 3–16, 1963.
- [Pit07] Nir Piterman. From nondeterministic Büchi and Streett automata to deterministic parity automata. *Logical Methods in Computer Science*, 3(3):5, 2007.
- [Rab69] Michael O. Rabin. Decidability of second order theories and automata on infinite trees. *Transaction of the AMS*, 141:1–35, 1969.
- [Saf89] Shmuel Safra. *Complexity of automata on infinite objects*. PhD thesis, Weizmann Institute of Science, 1989.
- [Saf92] Shmuel Safra. Exponential determinization for ω -automata with strong-fairness acceptance condition. In *Proc. 24th ACM Symp. on Theory of Computing*, 1992.

- [Sch09] Sven Schewe. Büchi complementation made tight. In *Proc. 26th Symp. on Theoretical Aspects of Computer Science*, volume 3 of *LIPICs*, pages 661–672. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany, 2009.
- [SN99] Helmut Seidl and Damien Niwiński. On distributive fixed-point expressions. *Theoretical Informatics and Applications*, 33(4–5):427–446, 1999.
- [Sta83] Ludwig Staiger. Finite-state omega-languages. *J. Comput. Syst. Sci.*, 27(3):434–448, 1983.
- [Str81] Robert S. Street. Propositional dynamic logic of looping and converse. Technical report, MIT, 1981. (The definition of Streett automata only appears in the technical report, not appearing in the conference paper with the same title.).
- [SV89] Shmuel Safra and Moshe Y. Vardi. On ω -automata and temporal logic. In *Proc. 21st ACM Symp. on Theory of Computing*, pages 127–137, 1989.
- [SV14] Sven Schewe and Thomas Varghese. Determinising parity automata. In *MFCS*, pages 486–498, 2014.
- [Tho90] Wolfgang Thomas. Automata on infinite objects. *Handbook of Theoretical Computer Science*, pages 133–191, 1990.
- [VW94] Moshe Y. Vardi and Pierre Wolper. Reasoning about infinite computations. *Information and Computation*, 115(1):1–37, 1994.
- [Yan08] Qiqi Yan. Lower bounds for complementation of omega-automata via the full automata technique. *Logical Methods in Computer Science*, 4(1), 2008.